

Quiz 4 – 9/20/2023

Instructions. You have 15 minutes to complete this quiz. You may use your plebe-issue TI-36X Pro calculator. You may not use any other materials.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

| Problem | Weight | Score |
|---------|--------|-------|
| 1a | 1 | |
| 1b | 1 | |
| 1c | 1 | |
| 2 | 1 | |
| Total | | / 40 |

Problem 1. Consider the Markov chain defined by the following one-step transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are two recurrent classes: $\mathcal{R}_1 = \{2, 3\}$ and $\mathcal{R}_2 = \{5\}$.

a. Classify each of the 5 states as transient or recurrent. No explanation necessary.

b. Suppose the Markov chain starts in state 4. What is the probability that the Markov chain is absorbed into state 5?

Here is the one-step transition probability matrix from the previous page, for your convenience:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- c. Suppose the Markov chain has reached state 2. What is the steady-state probability of being in state 3?

Problem 2. Consider a model of an elevator's movement from floor to floor in a high-rise building, in which the state of the system is defined as the floor on which the elevator is currently stopped, and the time step is defined to be the number of stops. Describe what assumptions need to be made in order for the Markov property to hold. (You do not need to discuss whether these assumptions are realistic.)